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MODELING OF THE FREE-SURFACE SHAPE IN LASER CUTTING OF METALS. 1. EFFECT OF POLARIZATION OF THE GAUSSIAN BEAM ON THE SHAPE OF THE SURFACE FORMED

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A three-dimensional problem of describing the shape of a surface formed owing to interaction of laser radiation with a substance in processes of laser cutting of metals is considered. The effect of radiation polarization (linear, elliptic, and circular) on the absorption factor is analyzed. For calculating the latter, a generalized formula is proposed, which takes into account the spatial orientation of the plane of incidence of radiation. The influence of laser-radiation parameters on the surface shape and cut depth is studied numerically. In the case of generation of a beam with the TEM_{00} *mode, it is shown that the use of elliptic polarization of radiation with a certain ratio of semi-axes, aligned with the beam direction, is preferable.*

Key words: *laser radiation, polarization, absorption, metal, gas-laser cutting, simulation.*

Introduction. The theory of failure of the material surface subjected to laser radiation [1–4] is based on the equation that describes the time evolution of the surface shape. Derivation of this equation is based on the condition of kinematic compatibility of surface points $\Phi(x, y, z, t) = 0$:

$$\frac{d\Phi}{dt} = \frac{\partial\Phi}{\partial t} + V_{\rm n}|\nabla\Phi| = 0, \qquad \boldsymbol{N} = \frac{\nabla\Phi}{|\nabla\Phi|} \tag{1}$$

($V_{\rm n}$ is the normal component of velocity of the surface and $\boldsymbol{N}$ is the normal to the surface).

The possibility of calculating the normal component of velocity of the surface points $V_{\rm n}$ in terms of the Poynting vector found by solving the Maxwell equations is discussed in [1]. Only the case of a small deviation of the sought surface from the sheet plane is considered, which allows approximate calculation of the Poynting vector. The absorption factor of the surface is calculated approximately; only transverse and longitudinal polarizations of radiation are considered.

In [2–4], $V_{\rm n}$ is determined from the local law of conservation of energy

$$V_{\rm n} = Q/L, \qquad L = c_s^0\rho_s^0(T_{\rm m} - T_0) + \rho_{\rm m}H_{\rm m}, \tag{2}$$

where Q is the power density of the incident radiation absorbed by a surface element, L is the energy of failure of a unit volume (this energy is equated to the energy necessary to heat the material from the room temperature T_0 to the melting point $T_{\rm m}$ and melt the material), ρ_s^0 and c_s^0 are the density and specific heat of the metal at the initial temperature T_0, $\rho_{\rm m}$ is the density at the melting point $T_{\rm m}$, and $H_{\rm m}$ is the specific melting heat. These notions are valid in the case of ideal removal of the liquid phase by the gas flow, where the thickness of the remaining liquid film is negligibly small.

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The surface shape obtained by laser cutting of metals with allowance for different polarizations of the beam was calculated in [3, 4] for thick materials (with a large ratio of the plate thickness to the cut width). It was noted that the limiting parameters of cutting by a beam with circular polarization are not worse than the corresponding parameters for linear polarization with absorption of the P wave in the front. The cutting efficiency can be increased by a factor of 1.5–2 by using radial polarization of the beam, which is obtained by superposing two mutually perpendicular TEM_{01} modes.

One can hardly agree with this estimate of efficiency of using radial polarization (as compared to circular polarization). The formulas used in [3, 4] to calculate the absorption factor are valid only for very thin sheet materials. Generation of high-quality TEM_{01} modes with different polarizations have not been adequately studied yet. Generation of the TEM_{01} mode inevitably leads to a decrease in radiation power and to an increase in the beam diameter and cut width.

The method of characteristics is used in [1–4] to solve Eqs. (1) and (2), which is not always convenient in practice because one has to numerically capture high gradients or discontinuities in the solutions obtained (in the case of thick materials).

The influence of the type of polarization of the beam on the absorption factor in the case of spatial interaction with an arbitrary material surface is analyzed in the present work.

Formulation of the Problem. The problem of laser-induced failure of the surface of opaque materials is complicated by the variety of interrelated physical processes, which cannot be described in detail at the moment [5]. The present formulation of the problem is based on the following assumptions.

1. The energy of absorbed radiation is spent only on material heating and melting. Evaporation and interaction of radiation with metal vapors are neglected.

2. It is assumed that hydrodynamic processes inside the cut (melt removal) under conditions of intense gas injection occur instantaneously; the thickness of the remaining liquid film is negligibly small.

3. The melted metal is removed by a neutral gas; hence, chemical reactions that occur in the case of oxygen-driven cutting are ignored.

4. Heat losses in the solid material are considered integrally; the cutting velocity is such that the locally one-dimensional distribution of temperature in the thin layer near the cut surface is valid.

5. The power of laser radiation is rather high, and the density of the absorbed power Q exceeds the threshold value for which formula (2) is valid.

6. The dependence of the absorption factor on temperature is ignored.

7. The material surface subjected to the action of radiation remains smooth. There is no regular roughness of the cut surface typical of real processes.

8. Absorption of reflected radiation is neglected; only single absorption of the beam is taken into account.

Based on these assumptions, the mathematical formulation of the problem on material-surface failure under the action of laser radiation reduces to the equation of kinematic compatibility of cut-surface points

$$\frac{\partial z_m}{\partial t} - V_{\mathrm{c}}\,\frac{\partial z_m}{\partial x} = -V_{\mathrm{n}}\sqrt{1 + \Big(\frac{\partial z_m}{\partial x}\Big)^2 + \Big(\frac{\partial z_m}{\partial y}\Big)^2}; \tag{3}$$

$$z_m(x, y, 0) = 0; \tag{4}$$

$$\frac{\partial z_m}{\partial x}(-\infty, y, t) = \frac{\partial z_m}{\partial x}(\infty, y, t) = 0, \qquad \frac{\partial z_m}{\partial y}(x, -\infty, t) = \frac{\partial z_m}{\partial y}(x, \infty, t) = 0, \tag{5}$$

where t is the time, x, y, and z are the spatial coordinates, $z = z_m(x, y, t)$ is the surface equation, and V_{c} is the velocity of beam motion (or cutting velocity) whose direction coincides with the Ox axis.

To calculate the normal component of surface velocity V_{n}, we use the local conservation law (2); an analog of this law with allowance for temperature dependences of material density and heat capacity was obtained in [5]:

$$V_{\mathrm{n}} = Q \Big/ \Big(\rho_{\mathrm{m}} H_{\mathrm{m}} + c_s^0 \rho_s^0 (T_{\mathrm{m}} - T_0) \int\limits_0^1 \nu(\bar{T})\, d\bar{T}\Big), \qquad Q = AI(x, y, z)\cos\gamma. \tag{6}$$

Here A is the absorption factor, $I(x, y, z)$ is the radiation intensity, and γ is the angle of incidence of the beam. The function $\nu(\bar{T})$, where $\bar{T} = (T - T_0)/(T_{\mathrm{m}} - T_0)$, takes into account the temperature dependence of the product of metal density and heat capacity [5].

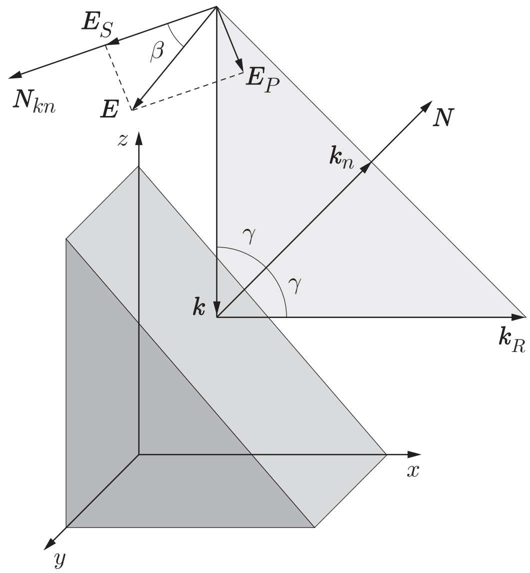

Fig. 1

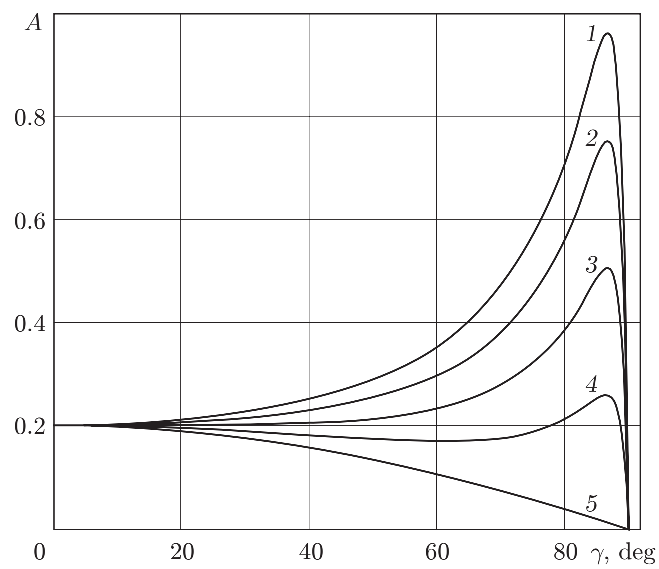

Fig. 2

Fig. 1. Interaction of the light beam with the surface element.

Fig. 2. Absorption factor versus the angle of incidence γ for $\beta = 90°$ (1), $60°$ (2), $45°$ (3), and $30°$ (4); curve 5 refers to $\beta = 0$.

We consider continuous radiation of a CO_2 laser with a wavelength $\lambda_0 = 10.6$ μm. The radiation intensity is described by the Gaussian distribution, which corresponds to the TEM_{00} mode [6]:

$$I(x, y, z) = \frac{2W}{\pi\omega_z^2} \exp\Big(-\frac{2r^2}{\omega_z^2}\Big), \qquad \omega_z = \sqrt{\omega_0^2 + \Big(\frac{(z - z_f)\lambda_0}{\pi\omega_0}\Big)^2}, \qquad r = \sqrt{x^2 + y^2}. \tag{7}$$

Here W is the radiation power, z_f is the distance from the plane $z = 0$ to the focal plane, and ω_0 is the beam radius in the focal plane.

The radiation reflection factors R_S and R_P are expressed by the Fresnel equations [7]:

$$R_S = \Big|\frac{\cos\gamma - (N_\omega^2 - \sin^2\gamma)^{1/2}}{\cos\gamma + (N_\omega^2 - \sin^2\gamma)^{1/2}}\Big|^2, \qquad R_P = \Big|\frac{N_\omega^2\cos\gamma - (N_\omega^2 - \sin^2\gamma)^{1/2}}{N_\omega^2\cos\gamma + (N_\omega^2 - \sin^2\gamma)^{1/2}}\Big|^2. \tag{8}$$

Here R_S and R_P are the reflection factors for the transverse (S) and longitudinal (P) radiation waves. The refraction factor $N_\omega = n_\omega + ik_\omega$ is a complex number (n_ω is the refraction index and k_ω is the material conductivity).

Linear Polarization of Radiation. We consider the case of linear polarization of the Gaussian beam. Figure 1 shows the surface element aligned at an angle γ to the axis Ox of the Cartesian coordinate system (x, y, z). The wave vectors of the incident radiation ($\boldsymbol{k}$) and reflected radiation ($\boldsymbol{k}_R$) and the unit vector normal to the surface $\boldsymbol{N}$ form the plane of incidence. The vector of the electric-field strength $\boldsymbol{E}$ is expanded into two projections. The reflection factors R_P and R_S correspond to the projection $\boldsymbol{E}_P$ in the plane of incidence and to the projection $\boldsymbol{E}_S$ normal to the plane of incidence. Let β be the angle between the vector $\boldsymbol{E}$ and the normal to the plane of incidence $\boldsymbol{N}_{kn} = \boldsymbol{N} \times \boldsymbol{k}/|\boldsymbol{k}|$, then $E_P = E\sin\beta$ and $E_S = E\cos\beta$. Since the direction of $\boldsymbol{k}$ coincides with the orth $\boldsymbol{e}_z$ (radiation is parallel to the axis Oz) and the vector $\boldsymbol{E}$ is perpendicular (or parallel) to beam motion, we have $\cos^2\beta = N_x^2$ (or $\cos^2\beta = N_y^2$), where N_x and N_y are the components of the vector normal to the surface. According to [8], the expression for the absorption factor is

$$A(\beta, \gamma) = 1 - R(\beta, \gamma) = 1 - I_r/I_0 = 1 - E_r^2/E^2$$
$$= 1 - (R_P E_P^2 + R_S E_S^2)/E^2 = 1 - R_S\cos^2\beta - R_P\sin^2\beta, \tag{9}$$

where I_r and I_0 are the intensities of reflected and incident radiation proportional to the squared strengths of the electric fields E_r and E, respectively.

Figure 2 shows the dependence of the absorption factor $A(\beta, \gamma)$. For $\beta = \pi/2$, the radiation is polarized parallel to the plane of incidence, which corresponds to the maximum absorption factor. As the angle β decreases, parallel polarization of the beam is monotonically transformed to perpendicular polarization, which corresponds to the minimum absorption factor for $\beta = 0$.

Elliptic Polarization of Radiation. We consider the case of elliptic polarization of the beam, where the end of the vector $\boldsymbol{E}$ in the plane (x, y) describes an ellipse with semi-axes a and b aligned parallel to the axes Ox and Oy. The relation $a^2 + b^2 = 1$ is satisfied thereby. We write the absorption factor in the form $A = a^2 A_x + b^2 A_y$, where A_x and A_y are the absorption factors for radiation linearly polarized parallel to the axes Ox and Oy. According to (9), we can write the formulas

$$
\begin{aligned}
A_x(\gamma, \beta_x) &= 1 - R_S(\gamma)\cos^2\beta_x - R_P(\gamma)\sin^2\beta_x, \\
A_y(\gamma, \beta_y) &= 1 - R_S(\gamma)\cos^2\beta_y - R_P(\gamma)\sin^2\beta_y,
\end{aligned}
\tag{10}
$$

where β_x and β_y are the angles between the normal to the plane of incidence $\boldsymbol{N}_{kn}$ and the axes Ox and Oy, respectively. The following equalities are valid for the angles β_x and β_y, respectively:

$$
\begin{aligned}
\cos^2\beta_x &= ((\boldsymbol{k}/|\boldsymbol{k}| \times \boldsymbol{N})\boldsymbol{e}_x)^2 = (\boldsymbol{N}, \boldsymbol{e}_y)^2 = N_y^2, \\
\cos^2\beta_y &= ((\boldsymbol{k}/|\boldsymbol{k}| \times \boldsymbol{N})\boldsymbol{e}_y)^2 = (\boldsymbol{N}, \boldsymbol{e}_x)^2 = N_x^2.
\end{aligned}
\tag{11}
$$

Substituting (11) into (10), we obtain the expression for the absorption factor in the case of elliptic polarization:

$$
A(\gamma, \boldsymbol{N}) = a^2 A_x + b^2 A_y = 1 - R_S(a^2 N_y^2 + b^2 N_x^2) - R_P(a^2(1 - N_y^2) + b^2(1 - N_x^2)). \tag{12}
$$

The absorption factor A depends strongly on the angle of incidence, spatial orientation of the normal vector to the surface, and radiation polarization, which is characterized by the semi-axes ratio $\xi = b/a$.

Circular Polarization of Radiation. For $a = b = 1/\sqrt{2}$, we have circular polarization of radiation. With allowance for the equality $N_x^2 + N_y^2 + N_z^2 = 1$, we obtain the following equation from (12):

$$
A_{\rm c} = 1 - [R_S(1 - N_z^2) + R_P(1 + N_z^2)]/2. \tag{13}
$$

Let the surface of the material being cut be such that the cut front and the side walls deviate little from the vertical. This is experimentally confirmed for thin sheets (of thickness $h = 1$–3 mm). In this case, the component N_z of the normal vector $\boldsymbol{N}$ is close to zero. Then, from (13), assuming that $N_z = 0$, we obtain $A_{\rm c} = 1 - 0.5(R_S + R_P)$. This is the known and widely used formula for approximate evaluation of the absorption factor in the case of circular polarization [1–5, 9–11].

Thus, relation (12) allow one to calculate the absorption factor for elliptical, circular, and linear polarizations of the beam in the general case of an arbitrary surface of the material.

Calculation Results and Their Discussion. Equation (3) with the initial conditions (4), boundary conditions (5), and closing relations (6)–(8) and (12) was solved numerically by a pseudo-transient method with an explicit finite-difference scheme with second-order approximation on a uniform grid. Within the framework of the problem posed, the cut surface is counted from the upper plane of the metal sheet ($z = 0$) to the limiting depth of material failure ($z < 0$). The major part of radiation interacting with metal is incident onto the cut surface at high angles. The main feature is the strong dependence of the absorption factor on the angle of incidence (see Fig. 2).

Figure 3 shows the projections of the cut shape in dimensionless variables in the plane (z, y) for linear and circular polarizations of the beam. The corresponding isolines of absorbed power $Q(x, y, z_m(x, y))$ in the plane (x, y) are plotted in Fig. 4. The dashed isoline corresponds to the beam radius $\omega_0 = 100$ μm. The dimensionless parameter $\sigma = 2W/[\pi\omega_0^2 V_{\rm c}(\rho_{\rm m} H_{\rm m} + \rho_s^0 c_s^0(T_{\rm m} - T_0))]$ characterizing the degree of the energy action on the material is 300, which corresponds to the power $W = 2$ kW and cutting velocity $V_{\rm c} = 44$ mm/sec.

In the case of cutting by S-polarized radiation ($\xi = \infty$), where the vector of electric-field strength is perpendicular to beam motion, the maximum of radiation absorption is located on the side walls (Fig. 4a); the absorption factor at the cut front is approximately the same as that on the side walls. The cut is wide, its surface is smooth, and its depth is approximately 1 mm (see Fig. 3a).

In the case of cutting by P-polarized radiation ($\xi = 0$), the vector $\boldsymbol{E}$ is parallel to beam motion (see Fig. 3b); the maximum density of absorbed power is at the cut front (see Fig. 4b) where the radiation is incident at an angle

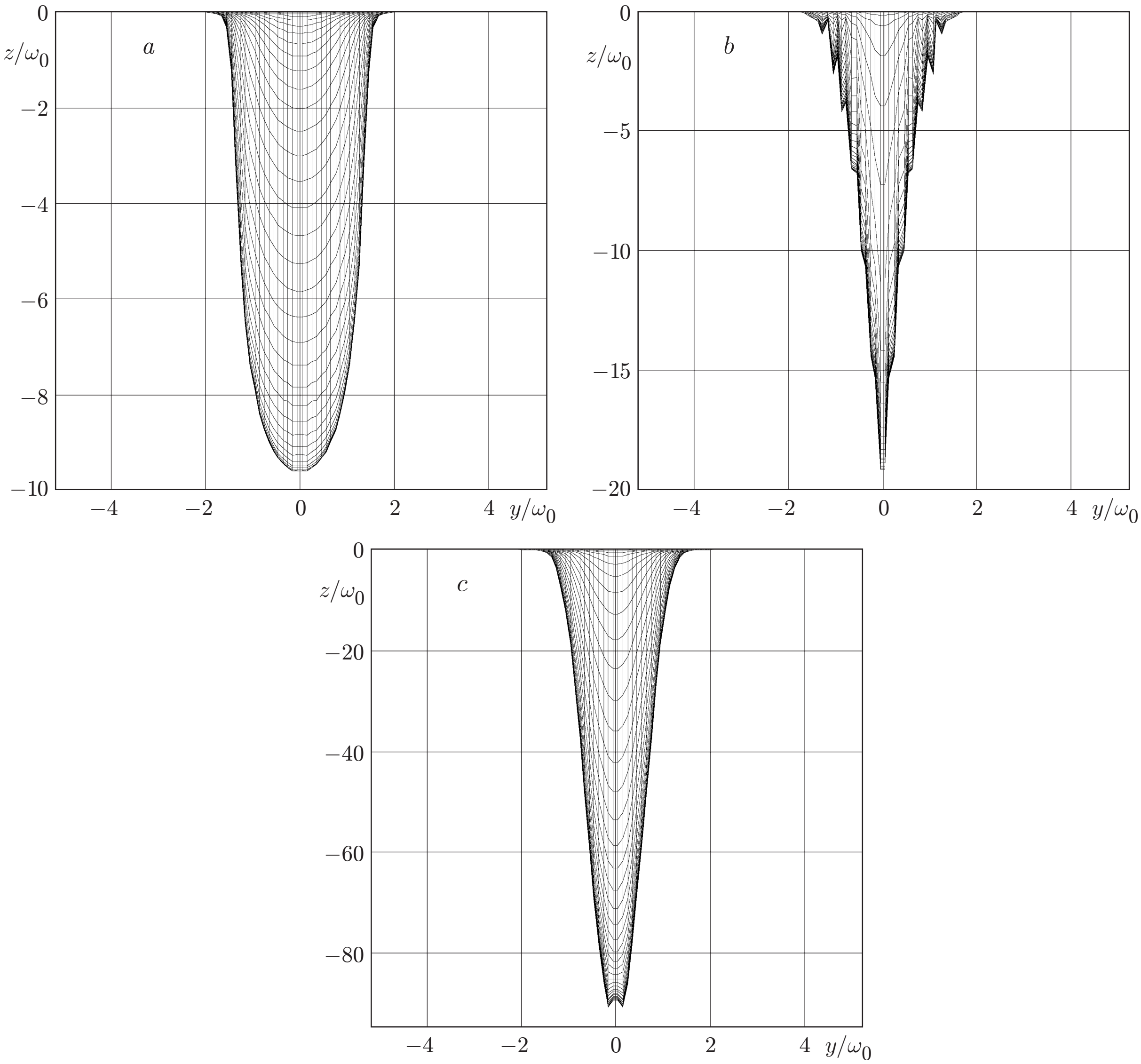

Fig. 3. Effect of the polarization type on the shape of the material surface: (a) linear polarization perpendicular to beam motion; (b) linear polarization parallel to beam motion; (c) circular polarization.

$\gamma \approx 85$–$87°$. The cut is narrow, and its maximum depth is about 2 mm because the major part of radiation does not penetrate inside the cut, being reflected on its front. Numerical nonuniformities of the surface, caused by a high gradient of absorbed power in the vicinity of the cut front, are clearly seen in Fig. 3b. To remove these nonuniformities, one has to use a numerical method of high-order accuracy.

In the case of circular polarization (see Fig. 3c), the absorbed radiation is distributed over the surface comparatively uniformly (see Fig. 4c); the shape of the side surface evolves to the vertical, which yields the maximum cut depth of 9 mm. In all cases, the irradiated surface tends to acquire a shape that ensures the minimum absorption of radiation.

Figure 5 shows the results of a series of numerical experiments on determining the maximum cut depth L as a function of the ratio b/a in dimensionless variables (L/ω_0 and b/a). The value of the dimensionless quantity σ was varied. It turned out that the maximum cut depth is much greater in the case of circular polarization ($b/a = 1$) than in the case of linear polarization ($b/a = 0, \infty$). The maximum on the curves corresponds to an elliptically polarized beam with the semi-axes ratio $b/a = 0.75$–0.80.

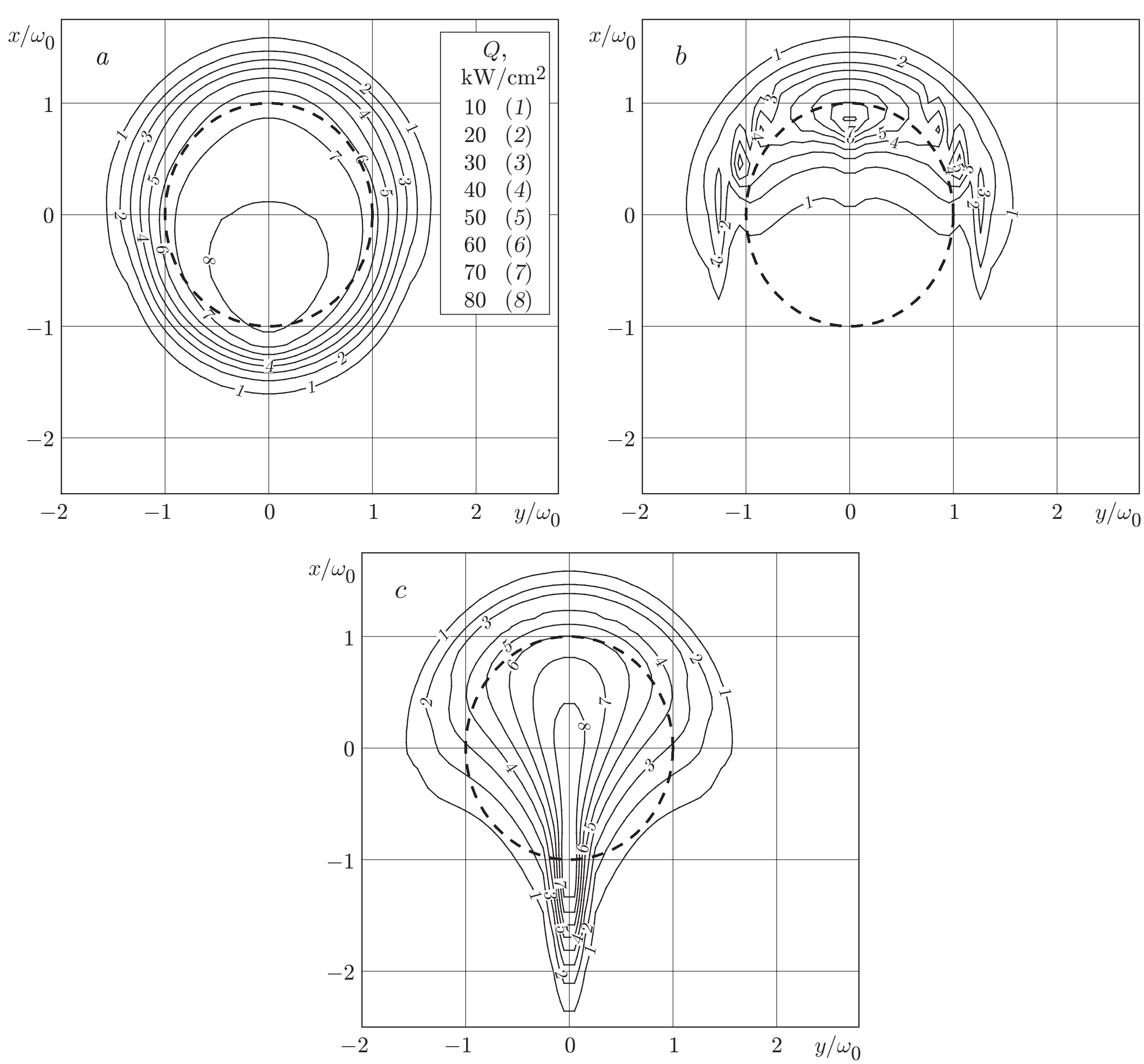

Fig. 4. Isolines of absorbed power $Q(x, y, z_m(x, y))$: (a) linear polarization perpendicular to beam motion; (b) linear polarization parallel to beam motion; (c) circular polarization.

Conclusions. The problem of describing the shape of the surface formed by laser cutting of metals by a powerful radiation flux with the TEM_{00} mode is considered. Cases with linear, elliptic, and circular polarizations of the beam are considered. A functional dependence (12) is proposed to calculate the absorption factor; this dependence takes into account the spatial orientation of the plane of incidence, which is important for cutting thick materials with a large ratio of the cut depth to the Gaussian beam diameter. Formula (12) allows one to calculate the absorption factor in the case of elliptical polarization of the beam with ellipticity oriented either perpendicular ($a < b$) or parallel ($a > b$) to beam motion. Depending on the ratio of ellipse semi-axes ($\xi = b/a$), the linear polarization (with absorption of the P wave for $\xi = 0$ or S wave for $\xi = \infty$ in the front) is monotonically transformed to elliptic polarization ($\xi < 1$, $\xi > 1$) and to circular polarization ($\xi = 1$). The effect of laser-radiation parameters on the surface shape and cut depth is studied numerically. It is shown that radiation with elliptic polarization with a certain ratio of semi-axes ($\xi \approx 0.75$), oriented along beam motion, is most efficient.

There is a common opinion in the literature (see, e.g., [9–11]) and among specialists on laser treatment of metals that the use of linearly polarized radiation with absorption of the P wave in the front is most efficient in practice. The present calculation results revealed the following (see Figs. 3–5). First, the efficiency of circular

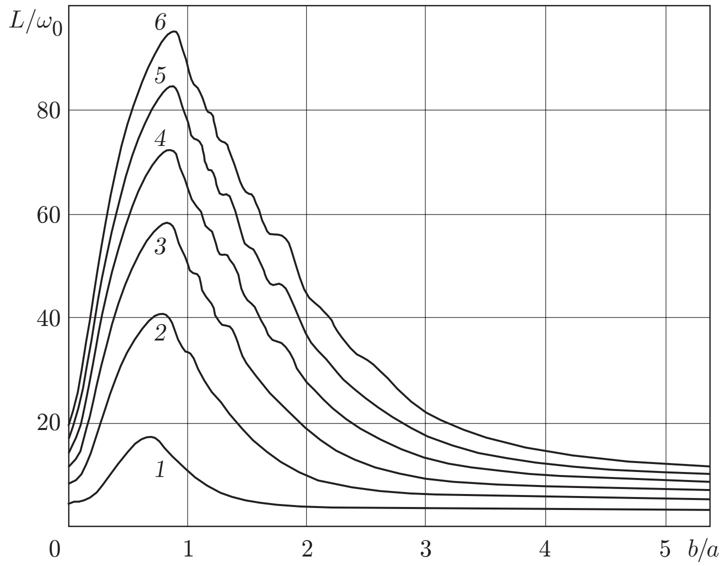

Fig. 5. Maximum cut depth L/ω_0 as a function of beam polarization b/a: $\sigma = 50$ (1), 100 (2), 150 (3), 200 (4), 250 (5), and 300 (6).

polarization of radiation is higher as compared to linear polarization (P wave), which agrees with the numerical data of [3–4] and with the analysis of available experiments [3]. Note that a beam with circular polarization is usually used in advanced technologies of laser cutting. The reason is the technical difficulties in controlling the plane of polarization of the electric-field vector in cutting elements of complicated configurations [10]. Second, the efficiency of elliptic polarization can be even higher than that of circular polarization. Therefore, straight-line cutting of sheet materials by an elliptically polarized beam is more beneficial in practice.

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REFERENCES

1. A. A. Vedenov, O. P. Ivanov, and A. L. Chernyakov, "Theory of destruction of the surface of opaque materials by laser radiation," *Kvant. Élektron.*, **11**, No. 12, 2397–2404 (1984).
2. G. P. Cherepanov and A. G. Cherepanov, "Shape and depth of cutting by a laser beam," *Fiz. Khim. Obrab. Mater.*, No. 2, 133–137 (1990).
3. A. V. Nesterov and V. G. Niz'ev, "Special features of cutting metals by a laser beam with axisymmetric polarization," *Izv. Ross. Akad. Nauk, Ser. Fiz.*, **63**, No. 10, 2039–2046 (1999).
4. V. G. Niziev and A. V. Nesterov, "Influence of beam polarization on laser cutting efficiency," *J. Phys. D, Appl. Phys.*, **32**, 1455–1461 (1999).
5. O. B. Kovalev, A. M. Orishich, V. M. Fomin, and V. B. Shulyat'ev, "Adjoint problems of mechanics of continuous media in gas-laser cutting of metals," *J. Appl. Mech. Tech. Phys.*, **42**, No. 6, 1014–1022 (2001).
6. A. N. Oraevskii, "Gaussian beams and optical resonators," *Tr. Fiz. Inst. Lebedeva*, **187**, 3–9 (1988).
7. E. Stuart and H. N. Rutt, "Selection criteria for polarizing mirrors for use in high-power CO_2-lasers," *J. Phys. D, Appl. Phys.*, **22**, 901–905 (1989).
8. M. Born and E. Wolf, *Principles of Optics*, Pergamon, Oxford (1975).
9. A. A. Vedenov and G. G. Gladush, *Physical Processes in Laser Processing of Materials* [in Russian], Énergoatomizdat, Moscow (1985).
10. A. G. Grigor'yants, *Fundamentals of Laser Treatment of Materials* [in Russian], Mashinostroenie, Moscow (1989).
11. J. Pawell, *CO_2 Laser Cutting*, Springer-Verlag, London (1998).